

Cubes of Binomials

$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)(x+y) \\&= (x^2+2xy+y^2)(x+y) \\&= x(x^2+2xy+y^2) + y(x^2+2xy+y^2) \\&= x^3+2x^2y+xy^2+x^2y+2xy^2+y^3 \\&= x^3+3x^2y+3xy^2+y^3\end{aligned}$$

(1)

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

(2)

$$\begin{aligned}\text{Ex) } (x+2)^3 &= x^3 + 3x^2(2) + 3x(2^2) + 2^3 \\&= x^3 + 6x^2 + 12x + 8\end{aligned}$$

$$\begin{aligned}\text{Ex) } (a-4b)^3 &= a^3 - 3a^2(4b) + 3a(4b)^2 - (4b)^3 \\&= a^3 - 12a^2b + 48ab^2 - 64b^3\end{aligned}$$

$$\begin{aligned}x^3+y^3 &= (x+y)^3 - 3x^2y - 3xy^2 \\&= (x+y)^3 - 3xy(x+y) \\&= (x+y)[(x+y)^2 - 3xy] \\&= (x+y)[x^2+2xy+y^2-3xy] \\&= (x+y)(x^2-xy+y^2)\end{aligned}$$

(3)

$$x^3-y^3 = (x-y)(x^2+xy+y^2)$$

(4)

Ex) Factor $8a^3 - 1$

$$x = 2a \quad y = 1$$

$$(2a - 1)((2a)^2 + (2a)(1) + (1)^2)$$
$$(2a - 1)(4a^2 + 2a + 1)$$

Ex) Factor $x^3 + 27y^3$

$$x = x \quad y = 3y$$

$$(x + 3y)(x^2 - (x)(3y) + (3y)^2)$$
$$(x + 3y)(x^2 - 3xy + 9y^2)$$