

Exam 3 Study Guide

Part I - Solving Systems of Equations

definitions

Inconsistent - A pair of equations that do not have a common solution

Dependent - Any solution of one equation is also a solution of the other.

Skills

- 1) Solve two variable systems of equations, simplification may be needed
- 2) Solve three variable system of equations
- 3) Solve combination of two and three variable equations

Part II - Solving Word Problems

formulas

$d = r \cdot t$ (distance = rate $\cdot$ time)

$a = s \cdot p$ (amount of substance = amount of solution $\cdot$ concentration)

Skills

- 1) Solve a distance rate and time problem (must consider units)
- 2) Solve a mixture problem
- 3) Solve a $d = rt$ problem where the distances are equal to each other

1a) Solve the system of equations:

$$\begin{cases} \frac{1}{2}(x-2) - \frac{1}{3}(y+3) = -2 & (1) \\ \frac{3(x-2)}{4} - \frac{5(y+1)}{2} = 2 & (2) \end{cases}$$

$$(1) \quad 6 \left[\frac{1}{2}(x-2) - \frac{1}{3}(y+3) \right] = [-2] 6$$

$$3(x-2) - 2(y+3) = -12$$

$$3x - 6 - 2y - 6 = -12$$

$$3x - 2y - 12 = -12$$

$$3x - 2y = 0$$

$$(2) \quad 4 \left[\frac{3(x-2)}{4} - \frac{5(y+1)}{2} \right] = [2] 4$$

$$3(x-2) - 10(y+1) = 8$$

$$3x - 6 - 10y - 10 = 8$$

$$3x - 10y - 16 = 8$$

$$3x - 10y = 24$$

$$3x - 2y = 0$$

$$-(3x - 10y = 24)$$

$$8y = -24$$

$$y = -3$$

$$3x - 2(-3) = 0$$

$$3x + 6 = 0$$

$$3x = -6$$

$$x = -2$$

$$\boxed{(-2, -3)}$$

2a)

$$\begin{cases} (1) & x - 2y + 3z = 2 \\ (2) & 2x - 3y + z = 1 \\ (3) & 3x - y + 2z = 9 \end{cases}$$

$$\begin{aligned} (1) & \quad 2x - 4y + 6z = 4 \quad (1) \\ - & \quad (2x - 3y + z = 1) \quad (2) \\ \hline & \quad -y + 5z = 3 \quad (4) \end{aligned}$$

$$\begin{aligned} (1) & \quad 3(2x - 3y + z) = (1)3 \quad (2) \\ - & \quad 2(3x - y + 2z) = (9)2 \quad (3) \end{aligned}$$

$$\begin{aligned} (5) & \quad 6x - 9y + 3z = 3 \quad (2) \\ - & \quad (6x - 2y + 4z = 18) \quad (3) \\ \hline & \quad -7y + z = -15 \quad (5) \end{aligned}$$

$$\begin{aligned} & \quad -7y + 35z = 21 \quad (4) \\ - & \quad (-7y + z = -15) \quad (5) \\ \hline & \quad 36z = 36 \end{aligned}$$

$$z = \frac{36}{36} = 1$$

$$\begin{aligned} & \quad -y + 5(1) = 3 \quad (4) \\ & \quad -y + 5 = 3 \\ & \quad -y = -2 \\ (6) & \quad y = 2 \end{aligned}$$

$$x - 2(2) + 3(1) = 2 \quad (1)$$

$$x - 4 + 3 = 2$$

$$x - 1 = 2$$

$$x = 3$$

$$\boxed{(3, 2, 1)}$$

$$3a) \begin{cases} x + 2y + 2z = 1 & (1) \\ 3y + 4z = 1 & (2) \\ 4x - y = 3 & (3) \end{cases}$$

$$\begin{aligned} 2(x + 2y + 2z) &= (1) \cdot 2 & (1) \\ 3y + 4z &= 1 & (2) \end{aligned}$$

$$\begin{aligned} 2x + 4y + 4z &= 2 & (1) \\ - (2) \quad 3y + 4z &= 1 & (2) \\ \hline 2x + y &= 1 & (4) \end{aligned}$$

$$\begin{aligned} (4) \quad 4x - y &= 3 & (3) \\ (8) + (4) \quad 2x + y &= 1 & (4) \\ \hline (8) \quad 6x &= 4 & \\ x &= 4/6 = 2/3 \end{aligned}$$

$$\begin{aligned} (2) \quad 4\left(\frac{2}{3}\right) - y &= 3 & (3) \\ \frac{8}{3} - y &= 3 \\ 8 - 3y &= 9 \\ -3y &= 1 \\ y &= -1/3 \end{aligned}$$

$$\begin{aligned} 3y + 4z &= 1 & (2) \\ 3(-1/3) + 4z &= 1 \\ -1 + 4z &= 1 \\ 4z &= 2 \\ z &= 2/4 = 1/2 \end{aligned}$$

$$\left(\frac{2}{3}, -\frac{1}{3}, \frac{1}{2}\right)$$

1b) A boy completes a $17\frac{1}{2}$ mile trip in $3\frac{1}{2}$ hr and 30 min, running part of the way at 8 mph and walking the rest at $4\frac{1}{2}$ mph. Find the number of miles walked.

	d	=	r	•	t
Ran	$8x$		8		x
Walked	$4\frac{1}{2}y$		$4\frac{1}{2} = \frac{9}{2}$		y
total	$17\frac{1}{2} = \frac{35}{2}$				$3\frac{1}{2} = \frac{7}{2}$

$$\begin{aligned} 8x + \frac{9}{2}y &= \frac{35}{2} \\ x + y &= \frac{7}{2} \rightarrow y = \frac{7}{2} - x \end{aligned}$$

$$8x + \frac{9}{2}\left(\frac{7}{2} - x\right) = \frac{35}{2}$$

$$8x + \frac{63}{2} - \frac{9x}{2} = \frac{35}{2}$$

$$32x + 63 - 18x = 70$$

$$14x + 63 = 70$$

$$14x = 7$$

$$x = \frac{7}{14} = \frac{1}{2}$$

(time ran)

$$8\left(\frac{1}{2}\right) = 4$$

(distance ran)

total distance

$$\frac{35}{2} - 4 \rightarrow \frac{35}{2} - \frac{8}{2} = \frac{27}{2} = 13\frac{1}{2}$$

$13\frac{1}{2}$ mi walked

2b) A chemist has two mixtures of a certain chemical, the first being a concentration of 80% and the second a concentration of 50%. How many c.c. of each chemical must be used to make a 60 c.c. mixture of a 72% concentration.

	A	=	S	P
C ₁	80/100 x		x	80% = 80/100
C ₂	50/100 y		y	50% = 50/100
M	72/100 · 60		60	72% = 72/100

$$\begin{array}{r} 72 \\ 4800 \\ - 4320 \\ \hline 480 \end{array}$$

$$\frac{80}{100}x + \frac{50}{100}y = \frac{4320}{100} \quad (1)$$

$$x + \frac{5}{6}y = 60 \quad (2)$$

$$80x + 50y = 4320 \quad (1)$$

$$x = 60 - \frac{5}{6}y \quad (2)$$

$$80(60 - \frac{5}{6}y) + 50y = 4320$$

$$4800 - 80y + 50y = 4320$$

$$-30y = -480$$

$$y = \frac{-480}{-30} = \frac{48}{3} = 16$$

$$\begin{array}{l} X = 60 - 16 \\ \boxed{X = 44} \end{array}$$

44 c.c. of C₁, 16 c.c. of C₂

3b) A man leaves a location driving 75 mph. A woman leaves the same location 30 minutes later driving 90 mph. If the man left at 4:00 pm, at what time will the woman catch up with him?

	d	$=$	r	$\cdot$	t
man	$75x$		75		x
woman	$90(x - \frac{1}{2})$		90		$x - \frac{1}{2}$

$$75x = 90(x - \frac{1}{2})$$

$$75x = 90x - 45$$

$$-15x = -45$$

$$x = 3 \text{ hr}$$

woman caught up with man at
7:00 pm