Systems of Equations - three Variables

1) Eliminate one variable from 2 equations.
2) Eliminate the same variable from a different 2 equations.
3) Use the resulting equations from step 1 and 2 to eliminate a second variable.
4) Substitute the solved variable back into one of the equations from step 1/step 2
5) We now have two solved variables, substitute both of them into one of the original equations.

Ex) Solve the system of equations:

$$\begin{cases} 4x+5y+t=0 & (1) \\ 8x-y+t=24 & (2) \\ 3x+2y+2t=1 & (3) \end{cases}$$

Step 1

$$\begin{cases} 4x+5y+t=0 & (1) \\ -(8x-y+t=24) & (2) \\ \hline -4x+6y=-24 & \\ -2x+3y=-12 & (4) \end{cases}$$

Step 2

$$\begin{cases} 2[8x-y+t=24]2 & (2) \\ 3x+2y+2t=1 & (3) \\ \\ 16x-2y+2t=48 & (2) \\ -(3x+2y+2t=1) & (3) \\ \hline 13x-4y=47 & (5) \end{cases}$$

Step 3

$$\begin{array}{r} {}^{2}47 \\ \times\ 3 \\ \hline 141 \end{array} \qquad \begin{array}{r} {}^{\circ}{}^{1}{}_{3} \\ \cancel{144}1 \\ -\ 48 \\ \hline 93 \end{array}$$

$$4[-2x+3y=-12]4 \quad (4)$$
$$3[13x-4y=47]3 \quad (5)$$

$$-8x+12y=-48$$
$$+\ 39x-12y=141$$
$$31x = 93$$
$$x = \frac{93}{31}$$
$$\boxed{x=3}$$

Step 4

$$-2(3)+3y=-12 \quad (4)$$
$$-6+3y=-12$$
$$3y=-6$$
$$\boxed{y=-2}$$

Step 5

$$4x+5y+t=0 \quad (1)$$
$$4(3)+5(-2)+t=0$$
$$12-10+t=0$$
$$2+t=0$$
$$\boxed{t=-2}$$

$$\boxed{(3,-2,-2)}$$